7. Significance of high velocity friction in dynamic rupture process

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Abstract. There are multiple lines of evidences discovered over the last couple of decades for the coseismic weakening of a fault which is much more drastic than what is predicted from the conventional friction laws verified at low slip rates. Such weakening undoubtedly affects the dynamic rupture propagation in various ways. For example, coseismic weakening considered in a framework of rate-weakening has been shown to play an important role in determining the manner of dynamic rupture propagation (crack-like versus pulse-like rupture) given a pre-stress level. Moreover in the sequence of earthquakes, the pre-stress is affected by the coseismic frictional resistance. In this article, some of the recent studies on the significance of high velocity friction shall be reviewed briefly.

1. Introduction

The frictional constitutive law of fault rocks is one of the main ingredients in considering dynamic rupture propagation during an earthquake, and thus actively being studied experimentally, theoretically, numerically,
and by field observations. The fault behavior is called “frictional” if the shear resistance $\tau$ is roughly proportional to the effective normal stress $\sigma_e$:

$$\tau = f \sigma_e = f \sigma - p$$

(1)

where $f$ is the friction coefficient, $\sigma$ is the total compressional normal stress on the fault, and $p$ is the pore pressure. There are so many studies on how $\tau$ varies during an earthquake, and one of the recent prominent experimental discoveries include remarkable reduction of $\tau$ (weakening) at coseismic slip rates ($> 0.1 \text{ m/s}$), sometimes referred to as “high slip rates” or “high velocity” [e.g., 1]. This article presents how such weakening affects the dynamic rupture propagation and its sequences.

Section 2 is devoted to describe the significance of the weakening at high slip rates in comparison with the typical observation of $\tau$ at lower slip rates. In order to say if the weakening is significance or not in terms of elastodynamics, one should compare the weakening rate, decrease in $\tau$ per unit increase in the slip rate $V$, of a fault with a characteristic weakening rate from elastodynamics. Section 3 describes such a comparison based on an assumption that the weakening is considered as a rate-weakening; there is a decreasing function $\tau(V)$. The effect of the significant rate-weakening is demonstrated by explaining some of the proposed criteria [2, 3] which determine the manner (crack-like vs. pulse-like) of rupture propagation [4].

The weakening at the coseismic slip rate affects not only coseismic, but also interseismic fault behaviors such as the level of the shear stress at which a fault operates. Section 4 presents a brief review of a recent study [5] which explicitly present this point by considering a coseismic increase in $p$ due to frictional heating (thermal pressurization of pore fluid) in a simulation of sequence of earthquakes.

2. Evidences of weakening of a fault at coseismic slip rates

Byerlee [6] compiled the data of friction experiments for variety of rocks, and concluded that the peak value of the frictional resistance $\tau_{\text{peak}}$, which is attained near the onset of sliding, is independent of the rock type with exception of clay minerals. At a compressional effective normal stress $\sigma_e$ below 200 MPa, $\tau_{\text{peak}}$ is given by $0.85\sigma_e$ with significant scattering of the data points, and at a higher normal stress, $\tau_{\text{peak}}$ is given by $50\text{MPa} + 0.6\sigma_e$. Thus, the peak friction coefficient $f_{\text{peak}}$ ranges from 0.6 to 0.85 independently of the rock type. This is so-called Byerlee’s law. This notion agrees with field observations such as the state of stress measured at boreholes [e.g., 7].
An earthquake occurs as a result of acceleration of relative motion (slip) on an active fault and its propagation. Then, the dependency of the sliding frictional resistance $\tau$ or the friction coefficient $f (= \tau / \sigma_e)$ on the speed of the relative motion (slip rate) $V$ or, more generally, the governing equation of $\tau$ is of great importance in considering the earthquake generation process. Dieterich [8, 9] investigated the dependency of $f_{\text{peak}}$ on the time of the stationary contact and the evolution of $f$ after a sudden change in $V$. $f_{\text{peak}}$ and $f$ respectively depend on the contact time and $V$ rather modestly; an $e$-fold increase in the contact time or $V$ causes a change in $f$ by on the order of 0.01 or smaller. Those experiments are typically carried out below 1 mm/s. Therefore, Byerlee’s law holds for the sliding frictional resistance $\tau$ as well as for $\tau_{\text{peak}}$ for a wide range of $\sigma_e$ and $V$ for variety of rocks. The value of $f$ from 0.6 to 0.85 (typically 0.7) has been referred to as a standard value of the friction coefficient of the rock. Note that during a large earthquake, an active fault slips at a slip rate on the order of 0.1 to 10 m/s. The experimental dataset which bases Byerlee’s law does not include experiments at such high slip rates.

Given a characteristic value of $f$ (~0.7) and a long-term slip rate of an active fault, one can calculate the frictional heating and the distribution of the heat flux at the surface. Lachenbruch and Sass [10] compared such theoretical prediction with measured heat flux along the San Andreas fault, California, and concluded that there is no detectable evidence of frictional heating. This observation suggests that the rate of frictional heating is much smaller than what is expected from the Byerlee’s law if $\sigma_e$ at the depth is assumed to be derived from lithostatic and hydrostatic stress condition. This issue is sometimes called as “heat flow paradox”, and the explanation for it has been one of the major tasks during last decades.

There are multiple lines of evidences for low frictional heating during coseismic slip. After 1999 Chi-Chi, Taiwan, Earthquake (Mw 7.6), for example, 2 direct measurements of temperature anomaly in bore holes which penetrate the Chelungpu fault, a source fault of Chi-Chi Earthquake, at about 300 m and 1100 m in depth revealed that $f$ during coseismic slip is about 0.1 if $\sigma_e$ is given by the lithostatic and hydrostatic condition [11, 12]. d’Alessio et al. [13] studied exhumed San Gabriel fault, California, and concluded that there is no evidence for frictional heating recorded as thermal healing of fission tracks, and estimated that $f$ must be below 0.4 for an earthquake which produces 4 m of slip.

Ancient earthquakes accompanied by high slip rates and resulting frictional heating can be recorded geologically as pseudotachylyte, melted and quenched glassy rock [e.g., 14]. Similar fused rocks are sometimes found in large landslides, and Erismann et al. [15] explained the occurrence of such
rocks as a result of frictional melting by producing pumice by friction experiments at a slip rate about 10 m/s. They reported that the averaged friction coefficient decreases with increasing normal stress from about 1 to as low as 0.17. There are many following studies producing pseudotachlyte in high velocity friction experiments [1, 16-25] as well as recent theoretical and numerical studies trying to formulate the shear resistance during fault motion [26-28]. Most of them reported much smaller f than characteristic value of Byerlee’s law especially at high normal stress, suggesting that melt lubrication is one of the strong candidates explaining the low frictional heating to some extent.

Pseudotachylyte is, however, rather rarely observed along exhumed faults which used to be located at a seismically active depth. Sibson [29] explained the scarcity of pseudotachylyte by hydro-thermal effects. Because the thermal expansivity of water is much larger than that of rocks, frictional heating causes an increase in the pore pressure in the shear zone and thus a decrease in σe and τ (see Eq. (1)) if the surrounding rock is impermeable enough so that it can confine the pressurized fluid efficiently. This mechanism is called thermal pressurization of pore fluid. It has been extensively studied mainly theoretically and numerically in terms of fault constitutive law, frictional instability, dynamic rupture propagation, and sequence of earthquakes [e.g., 3, 5, 29-46]. If a fault slips at a constant V forever with a finite value of f and if the onset of melting is neglected, then the shear resistance ultimately decays to zero with a length scale for slip depending on the hydrothermal properties and slip rate. Rempel and Rice [41] derived a condition for impossibility of onset of melting.

Tsutsumi and Shimamoto [1] conducted friction experiments for gabbro at V which ranges from about 5 cm/s to 1.3 m/s at σe = 1.5 MPa, and revealed that f dramatically decreases even without the generation of a melt layer and without any confinement of pore fluid (air); f is around 0.8 at 5 cm/s and 0.5 at 0.55 m/s. Later experimental studies have reconfirmed this observation for many different kinds of rocks in variety of experimental conditions (Figure 1, modified from Wibberley et al. [47]). The weakening is explained by different mechanisms for different rock types and different experimental conditions, such as localized temperature rise at the true area of contact (flash heating) [40, 48], macroscopic temperature rise and intrinsic temperature-weakening effect [49], formation of silica gel for SiO2-rich rocks [50], thermal decomposition of minerals and associated increase in pore pressure and/or generation of weak material (for example for coal [51], calcite [52], siderite [53], kaolinite [54], and gypsum [55]), and so on. Di Toro et al. [56] showed that the heat-induced weakening mechanisms can be distinguished by plotting the steady-state friction as a function of frictional power density.
Natural faults are not simple in terms of mineral composition, structure, and temperature-pressure and chemical conditions. Thus it is difficult to determine a physico-chemical mechanism which dominantly affects $\tau$ during dynamic rupture in a general case. But whatever the mechanism might be, it is likely that the shear resistance of a fault during coseismic slip is much smaller than what is predicted by Byerlee’s law and an ambient effective normal stress based on lithostatic and hydrostatic stress state. With some exceptional materials which has low friction coefficient at low slip rates (e.g., graphite [58]), there is a large difference between the shear resistances at low ($< 1$ mm/s) and coseismic ($> 0.1$ m/s) slip rates. This difference is associated with a much more remarkable rate-weakening than what is observed at the low slip rates (see Figure 1) if the assumption that the shear resistance $\tau$ is a function of the slip rate $V$ is appropriate.

It should be emphasized that at this point, the significance of the rate-weakening is defined in comparison with the conventional notion on the frictional resistance at low slip rates, and not relevant to the dynamic rupture process. In the next section, significance of the rate-weakening is discussed from the point of view of elastodynamics.

3. Significant rate-weakening in terms of elastodynamics

In the last section, significance of the rate-weakening at high slip rates is discussed in comparison with the frictional behavior at low slip rates ($< 1$ mm/s). In order to say that the absolute value of the slope of rate-
weakening [stress/velocity] is large or small in a context of dynamic rupture propagation, we need to consider nondimensionalization and normalization in a proper manner. In this section, a measure of rate-weakening is discussed in a dynamic rupture process.

Let us consider a rate-weakening frictional constitutive law of a fault,

\[ \tau = \tau(V) \quad \text{for} \quad V > 0 ; \quad d\tau / dV < 0 . \quad (2) \]

Cochard and Magariaga [58] conducted numerical simulations of dynamic rupture propagation for a rate-weakening fault. Many following studies [e.g., 2, 3, 58-60] employed remarkable rate-weakening by embedding it into friction laws which have one or more internal variables (state variables) and regularize Eq. (2). In those cases, the friction law can be written as

\[ \tau = \tau(V, \theta) \quad \text{(3)} \]

where \( \theta \) is the state variables which is a vector in general and evolves to a steady-state value \( \theta_{ss}(V) \) if \( V \) is fixed. The rate-weakening then means

\[ \tau_{ss}(V) = \tau(V, \theta_{ss}(V)) ; \quad d\tau_{ss} / dV < 0 . \quad (4) \]

Note that Rice et al. [61] proved that purely rate-weakening law can cause mathematical ill-posedness; there is no solution when Eq. (2) is coupled with elastodynamics if the rate-weakening is significant in a sense discussed in this section. Even though the regularization using a state variable is required in solving dynamic rupture problems, the notion of rate-weakening is still useful in discussing the results from those numerical studies as shown later.

\( \tau_{ss}(V) \) is usually assumed to be rather simple, having a value consistent with Byerlee’s law at \( V \) close to zero, much lower values at coseismic value of \( V \), and a transition between them (see Figure 1) corresponding to a maximum value of \(-d\tau_{ss}/dV,\)

\[ \chi \equiv \max(-d\tau_{ss}/dV) . \quad (5) \]

For example, Rice [40] explained the experimentally observed weakening before the onset of melting by flash heating, and proposed a friction law,

\[ \tau_{ss} = f_{ss}\sigma_e = \begin{cases} f_{LV}\sigma_e & V \leq V_w \\ \left(V_w/V, f_{LV} - f_w + f_w\right)\sigma_e & V \geq V_w \end{cases} \quad (6) \]
where \( f_{ss} \) is the steady-state friction coefficient, \( f_{LV} \) is the steady-state friction coefficient at low slip rates, \( V_w \) is the slip rate at which the weakening at high slip rate becomes efficient, and \( f_w \) is the friction coefficient at high enough slip rates (Figure 2). In this case, the maximum slope of the rate weakening is achieved at \( V = V_w \):

\[
\chi = \frac{f_{LV}(V_w) - f_w}{V_w} \sigma_e .
\]  

(7)

A question is how to evaluate \( \chi \) in terms of elastodynamics.

The dynamic rupture process is often considered as a problem dealing with an interaction between a boundary (fault) on which a rupture propagates and surrounding elastic medium with inertial effects. \( \chi \) is a quantity having a dimension of \([\text{stress/velocity}]\), and there is a scale for it given by elastodynamics. Elastodynamics has 3 physical properties, shear modulus \( \mu \), Poisson’s ratio \( \nu \), and density \( \rho \). Then \((\mu \rho)^{1/2} = \rho c_s = \mu / c_s \) (acoustic impedance of s-wave) is a characteristic value of the medium having a dimension of \([\text{stress/velocity}]\), where \( c_s \) is the s-wave speed which is equal to \((\mu / \rho)^{1/2}\). Therefore, it is reasonable to measure \( \chi \) nondimensionalized by a quantity which is proportional to the impedance.

Let us consider a planer fault embedded in a linearly elastic full space which is homogeneous. The comparison of \( \chi \) and the shear acoustic impedance is most visible in a boundary integral expression for the traction on the fault, \( \tau \) [e.g., 58, 62-65],

\[
\tau(r,t) = \tau_0(r,t) + \varphi[V; r, t] - \eta \cdot V(r,t) ,
\]  

(8)

where \( r \) is the position vector which spans on the fault, \( t \) is time, \( \tau_0 \) is the traction on the fault if there is no slip on the fault, \( \varphi \) is a convolution term which depends on the history of the jump in the particle velocity across the fault \( V \). If a unit normal vector to the fault is denoted as \( n \) and the other Cartesian basis vectors are as \( p_1 \) and \( p_2 \), then a second rank tensor \( \eta \) can be written as,

\[
\eta = \frac{\rho c_p}{2} nn + \frac{\rho c_s}{2} p_1 p_1 + \frac{\rho c_s}{2} p_2 p_2 = \frac{\mu c_p}{2 c_s^2} nn + \frac{\mu}{2 c_s} p_1 p_1 + \frac{\mu}{2 c_s} p_2 p_2 ,
\]  

(9)

where \( c_p \) is the p-wave speed and \( \rho c_p \) is the acoustic impedance of p-wave. The third term in Eq. (8) is called as the radiation damping term [66]. In a 2-dimensional problem without allowing opening of a fault, we have only to consider one component of Eq. (8) in the shear direction,
Figure 2. Friction coefficient \( f_{ss} \) as a function of slip rate \( V \) predicted by Eq. (6) which accounts for flash heating of microscopic asperities [40].

\[
\tau(x,t) = \tau_0(x,t) + \phi[V;x,t] - \eta V(x,t) \quad \eta = \frac{\mu}{2c_s},
\]

(10)

where \( x \) is the spatial coordinate along the fault. \( \eta \) shall be chosen as the scale having the dimension of [stress/velocity]. The radiation damping term takes care of the instantaneous effect between \( V \) and \( \tau \). This term can be understood as a linear interpolation between stress-free and glued boundary conditions. Note that if the fault is a surface of constant stress (i.e., \( \tau = \tau_0 \) where \( \tau_0 \) is constant with time) and the “stress wave” \( \phi \) propagates on the fault, then the propagating distribution of the slip rate \( V \) is proportional to \( \phi \) as discussed by Brune [67].

In order to satisfy the boundary condition on the fault, Eqs. (2) or (3) and (10) must be satisfied simultaneously. For simplicity, we use the concept of pure rate-weakening in the discussion here assuming that the state evolution is rapid enough compared with the evolution of \( \phi \). The significance of \( \chi/\eta \) is evident as indicated in Figure 3. Let us consider a loading history \( \phi(t) \) at a certain point on a fault such that the point on the fault is initially locked (\( V = 0 \)), ruptured and slipped at coseismic slip rate, and decelerated and relocked at the final condition. For illustration purpose, the initial and final value of \( \phi \) us chosen to be equal to each other in Figure 3. As \( \phi \) varies continuously with time \( t \), \( \chi/\eta < 1 \) results in continuous change in \( V \) (Figure 3a) while \( \chi/\eta < 1 \) causes abrupt jumps in \( V \) (Figure 3b) as discussed by Cochard and Madariaga [58]. When a point on the fault stop sliding as \( \phi \) decreases, \( \chi/\eta < 1 \) causes an efficient brake which is turned on at \( V = V_{\text{pulse}} \),
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Figure 3. Trajectories of \((V, \tau)\) at a point on a fault which is governed by a purely rate-weakening friction law. The convolutional term \(\varphi\) increases to a peak value and decreases to the initial value for simplicity. (a) The case with \(\chi/\eta < 1\); the rate-weakening is not significant in terms of elastodynamics. (b) The case with \(\chi/\eta > 1\); the rate-weakening is significant in terms of elastodynamics. Significant rate-weakening defined by \(\chi/\eta > 1\) causes efficient break at the deceleration at \(V = V_{\text{pulse}}\).

the slip rate at which \(\tau_{ss}(V)\) and the line which has a slope of \(-\eta\) fit tangentially as shown in Figure 3b. With a rate- and state-dependent friction law, at this point, the trajectory of \((V, \tau)\) diverges from \(\tau_{ss}(V)\), and \(V\) starts decreasing towards zero at \(\tau\) much smaller than \(\tau_{ss}(V)\) unless a loading is applied by incoming wave to this point (i.e., an increase in \(\varphi\)) [e.g., 3]. This
behavior causes a qualitative difference in the manner of dynamic rupture propagation.

3.1. Self-healing pulse-like rupture due to significant rate-weakening

Heaton [4] have revealed that many earthquakes occur in a self-healing pulse-like manner rather than a crack-like manner; slip on a fault propagates as a localized pulse of slip rate which has a rupture front and a healing front (Figure 4). The efficient brake due to the significant rate-weakening in terms of elastodynamics, $\chi/\eta > 1$, plays an important role in determining whether a rupture propagates in crack-like or a pulse-like manners [2,3,58-60, 68-70]. Note that there are other mechanisms proposed which cause the generation of the pulse-like rupture propagation such as a contrast in material properties across the fault [e.g., 71], and arrest waves from fault edges or from heterogeneity along the fault [e.g., 72-75], but those mechanisms will not be discussed here. In following subsections, some of the proposed criteria determining the manner of rupture propagation shall be presented which help understanding of the transition between crack-like and pulse-like ruptures and significance of the coseismic weakening.

Figure 4. A schematic diagram showing a crack-like (upper) a pulse-like (lower) ruptures.
3.1.1. Impossibility of crack-like rupture at low background shear stress

Zheng and Rice [2] derived a sufficient condition for non-existence of crack-like ruptures. Please see their paper for detailed derivation. For a planer fault, the functional term $\varphi$ in Eq. (10) satisfies

$$\int_{-\infty}^{\infty} \varphi(x,t) dx = 0 \cdot$$ (11)

Note that the rupture is a process of re-distribution of shear stress on a fault, and not a process of stress drop if one considers the entire system such as an infinitely long fault in a two-dimensional problem. Zheng and Rice [2] mathematically proved the following. Suppose there is an expanding crack-like rupture $S_{\text{rupt}}(t)$ which has a symmetry around $x = 0$. Then the shear stress is concentrated outside it $S_{\text{out}}(t)$,

$$\int_{S_{\text{out}}(t)} \varphi(x,t) dx \geq 0 \cdot$$ (12)

Eqs. (11) and (12) immediately yield

$$\int_{S_{\text{in}}(t)} \varphi(x,t) dx < 0 \cdot$$ (13)

The spatial integral of Eq. (10) inside $S_{\text{rupt}}(t)$ is then expressed as

$$\int_{S_{\text{rupt}}(t)} \tau - (\tau_0 - \eta V) dx = \int_{S_{\text{rupt}}(t)} \varphi dx < 0 \cdot$$ (14)

Assuming that $\tau = \tau_{ss}(V)$, if

$$\tau_{ss}(V) - (\tau_b - \eta V) \geq 0 \text{ for all } V > 0,$$ (15)

where $\tau_b$ is background shear stress, then it is impossible for Eq. (14) to be satisfied. Therefore, a crack-like rupture can not exist at such a low $\tau_b$. Note that $\tau_0$ is the spatial average of $\tau_0$ inside $S_{\text{rupt}}$, and approaches to the spatial average of $\tau_0$ over the entire fault as the hypothetical crack-like rupture expands.

This theorem rigorously gives a sufficient condition for the non-existence of the crack-like rupture. Let us define a critical shear stress value, $\tau_{\text{pulse}}$,
which is defined by the intersection of $V = 0$ and a line having the slope of $-\eta$ which tangentially fit to $\tau_{ss}(V)$ (Figure 3b). Then Eq. (15) is equivalent to

$$\tau_b < \tau_{pulse}. \quad (16)$$

If the background shear stress $\tau_b$ is smaller than $\tau_{pulse}$, then an expanding crack-like rupture is impossible.

If a rupture is initiated by a compact over-stressed region which has high $\tau_0$, the ruptures is crack-like just after its initiation, and it may undergo a transition to a pulse-like rupture or be arrested. Those transitions can understood through this theorem. $\tau_b$, a spatial average of $\tau_0$ over $S_{rupt}$, will decrease with $S_{rupt}$ expands. Even if $\tau_b$ is larger than $\tau_{pulse}$ while $S_{rupt}$ is small and thus a crack-like rupture is possible while the rupture is small, $\tau_b$ can become smaller than $\tau_{pulse}$ for large enough hypothetical ruptured area. In this case, the rupture must become pulse-like or arrested before it expands to such a size.

Zheng and Rice [2] conducted numerical simulations with a rate- and state-dependent friction law and verified that this theorem works even if there are process zones within which the purely rate-weakening law is not a good approximation. Also, they showed that pulse-like ruptures exist at a $\tau_b$ larger than, but close enough to $\tau_{pulse}$. Similar results have been obtained by following studies [e.g., 3, 70, 71].

### 3.1.2. Condition in terms of dynamic stress drop

Noda et al., [3] have conducted dynamic rupture simulations accounting for flash heating (Eq. (6)) for two-dimensional anti-plane cases, and explained the critical value of the background shear stress $\tau_b$ between pulse-like and crack-like ruptures by considering a first-order approximation of the friction law in a necessary and sufficient manner.

The idealization of the friction law is the following. The shear resistance during coseismic sliding, $V > V_{pulse}$, is not significantly dependent on $V$. Therefore, the frictional resistance at this slip rate regime could be regarded as constant at $\tau_d$ (Figure 5a). On the other hand, the value of $\tau_{ss}$ in the slip rate regime where the rate-weakening is significant does not matter much. In fact, in the purely rate-weakening case, $\tau_{ss}$ at slip rates where $|d\tau_{ss}/dV| > \eta$ does not affect the solution at all (Figure 3b). There is a slip rate ($V_{pulse}$) at which the efficient braking on the fault slip takes place (Figure 5a) between those two regimes. In this idealized friction law, $0 < V < V_{pulse}$ is a forbidden range for the slip rate.

Let us hypothetically consider a singular anti-plane crack expanding self-similar mannerly. The shear stress is initially uniform at $\tau_b$, and a ruptured
Figure 5. (a) Idealized friction law used in discussing the criterion between crack-like and pulse-like ruptures. At high enough slip rate $V > V_{\text{pulse}}$, the rate-weakening is not significant and thus the shear stress is idealized as a constant. (b) A possible self-similar crack-like rupture. $V_\infty > V_{\text{pulse}}$. (c) A self-similar crack-like rupture which is impossible because of the forbidden slip rate range, $0 < V < V_{\text{pulse}}$. 
region propagates bilaterally by a constant rupture speed $V_r$ in which the shear stress is uniformly equal to the kinetic frictional resistance, $\tau_d = \tau_b + \Delta \tau$. The corresponding slip rate distribution is [76, 77]

$$V = \text{Re} \left[ F \left( \frac{V_r}{c_s} \right) \frac{-\Delta \tau}{\mu} \frac{V_r}{\sqrt{1-(x/V_r t)^2}} \right],$$

where $F(V_r/c_s)$ is a nondimensional factor on the order of unity for realistic $V_r/c_s$ (Figure 5b). This solution is valid if the range of $V$ does not intersect with the forbidden slip rate range $(0, V_{\text{pulse}})$. At a point on the fault except $x = 0$, $V$ is zero before the rupture reaches there, infinitely large at the rupture front, and decrease towards a certain value $V_\infty$ which is given by

$$V_\infty = \lim_{t \to \infty} V = F \left( \frac{V_r}{c_s} \right) \frac{-\Delta \tau}{\mu} V_r.$$  

(17)

If the fault is governed by the idealized friction law (Figure 5a), this hypothetical crack-like rupture is possible if and only if $V_{\text{pulse}} \leq V_\infty$ (Figure 5b and c). Noda et al. [3] discussed their numerical results along this line after a rough estimate of $F(V_r/c_s) \cdot V_r/c_s$ and $\Delta \tau$.

Let us consider the comparison with the condition by Zheng and Rice [2] in the idealized friction law discussed in this section (Figure 5a). The threshold of $\tau_b$ for impossibility of the crack-like rupture, $\tau_{\text{pulse}}$, is given by

$$\tau_{\text{pulse}} = \tau_d + \eta V_{\text{pulse}},$$

(19)

where $\tau_d$ is the shear resistance at $V > V_{\text{pulse}}$. The critical value of $\tau_b$, $\tau_{bc}$ obtained from $V_{\text{pulse}} = V_\infty$ is given by

$$\tau_{bc} = \tau_d + \eta V_{\text{pulse}} \times 2 \left( F \left( \frac{V_r}{c_s} \right) \frac{V_r}{c_s} \right)^{-1} \equiv \tau_d + \eta V_{\text{pulse}} \Xi \left( \frac{V_r}{c_s} \right).$$

(20)

The critical value of the dynamic stress drop $-\Delta \tau$ is given by $\eta V_{\text{pulse}} \Xi(V_r/c_s)$. Figure 6 shows $\Xi(V_r/c_s) > 1$, indicating $\tau_{bc} > \tau_{\text{pulse}}$. Therefore, the criterion by Noda et al. [3], which is necessary and sufficient, is consistent with and more restrictive than that by Zheng and Rice [2] in this specific problem although the former requires idealization in order to be applied to a general case which causes ambiguity.

It should be emphasized that the critical value of $-\Delta \tau$, $\eta V_{\text{pulse}} \Xi(V_r/c_s)$ could be constrained from physical properties of the fault and surrounding
medium, based on an assumption on the rupture speed. In the case numerically studied by Noda et al. [3], $V_{\text{pulse}} = 1.5$ m/s (at $\sigma_e = 126$ MPa corresponding to 7 km depth) and $\eta = \mu/2c_s = 5$ MPa(m/s)$^{-1}$. For a rupture which propagates at a speed about $V_r/c_s \approx 0.8$, $\Xi(V_r/c_s)$ is about 2 (Figure 6) and then the critical value of $-\Delta\tau$ becomes about 15 MPa. This estimation explains their numerical results very well (see appendix C in their paper).

4. Effect of coseismic weakening on the long-term fault behavior

In the previous section, the effect of coseismic weakening on dynamic rupture is discussed with the background shear stress as one of the parameters. However, the background shear stress or a pre-stress before each dynamic event is determined through the history of the fault behavior which includes sequence of earthquakes and interseismic fault motion. Undoubtedly, the coseismic weakening affects the shear stress right after an earthquake, during the following interseismic period, and the pre-stress before the next dynamic event. Therefore, the drastic coseismic weakening probably affects the characteristics of the long-term fault behavior such as the overall stress level at which the active fault operates. This section presents a review of a recent study by Noda and Lapusta [5] which investigated the effect of coseismic increase in the pore pressure $p$ due to frictional heating (thermal pressurization of pore fluid) in a sequence of earthquakes.
4.1. Hydrothermal effects on frictional resistance

In Section 3, the weakening at coseismic slip rates has been discussed in a framework of rate-weakening, a decreasing function $\tau(V)$ or $\tau_{ss}(V)$. There are many possible fault constitutive laws which do not have those straightforward approximations by functions of $V$ but produce weakening at coseismic slip rates. Among them, thermal pressurization of pore fluid is discussed in this section. This mechanism is one of the best studied weakening mechanisms [e.g., 3, 5, 29-46].

Figure 7 is a schematic diagram of thermal pressurization of pore fluid. Rapid sliding of a fault during an earthquake produces frictional heating which causes an increase in the temperature $T$ by the fault. If the surrounding fault rock which is porous material is saturated with water, both rock and water try to expand thermally. The thermal expansion coefficient of water is much larger than that of rocks [e.g., 40, 41]. If the rock is not permeable enough, the pressurized water is confined near the shear zone and fluid pressure $p$ increases locally, causing a decrease in the effective normal stress $\sigma_e$ and thus dynamic weakening during an earthquake (see Eq. (1)).

Thermal pressurization of pore fluid is often modeled by considering diffusion of $T$ and $p$ normal to the fault with source terms corresponding to frictional heating [e.g., 30],

$$\frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial y^2} + \frac{\omega}{\rho c},$$

Figure 7. Schematic diagram showing the process of thermal pressurization of pore fluid. This figure is from Noda and Lapusta [5].
where \( y \) is the spatial coordinate normal to the fault, \( \alpha_{th} \) and \( \alpha_{hy} \) are thermal and hydraulic diffusivities, respectively, \( \rho_c \) is the specific heat capacity of the medium, \( \Lambda \) is the increase in \( p \) per unit increase in \( T \) under undrained condition, and \( \omega \) is distribution of frictional heat generation which satisfies

\[
\int_{-\infty}^{\infty} \omega \, dy = \tau V = (\sigma - p\big|_{y=0}) fV
\]

if all of the dissipated energy by frictional resistance turns into heat. The hydraulic diffusivity \( \alpha_{hy} \) is given by \( k/\eta_w \beta \) where \( k \) is permeability of the rock, \( \eta_w \) is water viscosity, and \( \beta \) is storage capacity. Note that the nonlinear terms such as convection and heat generation due to pressurizing the fluid are neglected here. For the full formulation, see Mase and Smith [32]. For materials with low enough permeability (\( k < 10^{-16} \text{ m}^2 \)), the convective term is not important [30, 32, 33]. Vredevoogd et al. [78] conducted numerical simulations that included all terms in the conservation equations [32], and demonstrated that the nonlinear terms may be safely neglected.

Among the physical properties which appear in Eqs. (21) and (22), the hydraulic diffusivity \( \alpha_{hy} \) has the largest variation for orders of magnitude, depending on the rock types, effective mean stress, chemistry of pore water, and so on [e.g., 36, 79-88]. Thermal pressurization is efficient for mature faults with well-developed fault core which has low \( k \) and thus \( \alpha_{hy} \). Figure 8 (Figure 11 in Wibberley and Shimamoto [85]) shows an example of the internal and permeability structures of mature faults (Median Tectonic Line, Southwest Japan, as an example studied by Wibberley and Shimamoto [85]). Fracturing of an intact rock (e.g., mylonites and metapelitic schist in Figure 8) remarkably increases permeability \( k \) and thus \( \alpha_{hy} \) locally. As cataclastic deformation accumulates towards the fault core, the fault rock becomes granulated (fault gouge), and then grain size reduction due to further cataclastic deformation causes a decrease in the permeability near the central slipping plane in Figure 8 which is also called as “principal slip surface” [40]. Mature faults which host large earthquakes have material with low permeability by the principal slip surface where coseismic shear strain is localized.

The width of the distribution of shear strain rate \( w \) plays an important role in thermal pressurization. The amplitude of heat generation density is
Figure 8. An example of (a) internal and (b) permeability structure in the mature fault zone (Median Tectonic Line, Southwest Japan as an example). This figure is Figure 11 in Wibberley and Shimamoto [85]. Permeability was measured with nitrogen as a pore fluid at different confining pressure, 50 MPa, 100 MPa, and 200 MPa, with 20 MPa of pore pressure.

Inversely proportional to $w$, and the system can be approximated by adiabatic and/or undrained limits if the diffusion lengths of $T$ and/or $p$ are much smaller than $w$. Observation of the exhumed fault and drilled fault core [89-92], and samples after rotary shear friction experiments [92, 93] suggest that the shear deformation localizes within a very thin layer typically sub-millimeters thick which are often recognized as the layer of preferred orientation of platy minerals.

Rice [40] derived an analytic solution to Eqs. (21) and (22) at fixed $V$ and $f$ for a shear zone localized to a mathematical plane,

$$\omega = (\sigma - p) f V \delta_d(y)$$

(24)
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where $\delta_D$ is Dirac’s delta function. The analytic solution for the frictional resistance is written as

$$\tau = (\sigma - p_0) f \exp\left(\frac{\delta}{L_0}\right) \text{erfc}\left(\sqrt{\frac{\delta}{L_0}}\right),$$  \hspace{1cm} (25)

where $\sigma$ is the total normal stress, $p_0$ is the initial pore pressure, $\delta$ is slip on the fault, and $L_0$ is a length scale given by

$$L_0 = \frac{4}{f^2} \left(\frac{\rho c}{\Lambda}\right)^2 \frac{\sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}}}{V}.$$  \hspace{1cm} (26)

For a range of realistic set of parameters, Rice [40] estimated $L_0$ as 2 to 50 mm at coseismic slip rates. Figure 9 shows the evolution of $\tau$ predicted by Eq. (25) (modified from Figure 3 in Rice [40]). It should be emphasized that the apparent length scale of the displacement required for weakening of the fault depends on the final slip of the event. Rice [40] successfully explained the dependency of the seismic fracture energy, the area below $\tau$ as a function of slip and above the final value of $\tau$, on the size of the earthquake [94] using those stress-reduction curves in Figure 9. The frictional resistance keeps decreasing towards zero as the fault slips at a constant $V$. Although it is true that the weakening due to thermal pressurization is more efficient at higher $V$ (i.e., $L_0$ decreases with $V$), the concept of rate-weakening becomes ambiguous.

Andrews [34] conducted dynamic rupture simulation with slip-weakening friction law for $f$ and thermal pressurization which produced crack-like ruptures, and pointed out that efficient thermal pressurization produces nearly complete stress drop.

![Figure 9.](image-url) Figure 9. Figure 3 in Rice [40]. Analytic solution to thermal pressurization with a deformation localized on a mathematical plane, Eq (25), is plotted with different horizontal scales.Apparently, the slip required for the weakening of a fault is always a good fraction of the total slip.
4.2. Effect of coseismic weakening on the sequence of earthquakes

Noda and Lapusta [5] developed a suitable methodology to implement Eq. (21) and (22) in a calculation of sequence of earthquakes which accounts for full inertial effect in coseismic periods and long-term tectonic loading [64, 65], and examined the effect of coseismic weakening. This subsection presents a brief review of their study. For detailed methodology and results, please refer to their paper.

They considered a fault which is governed by a rate- and state-dependent law,

\[ f = a \sinh^{-1} \left[ \frac{V}{2V_0} \exp \left( \frac{f_0 + b \ln(V_0 \theta / L)}{a} \right) \right] \approx f_0 + a \ln(V/V_0) + b \ln(V_0 \theta / L). \] (27)

The \( \sinh^{-1} \) regularization is important only if the shear stress on the fault becomes so small that \( f \) is close to or smaller than \( a \). For the physical basis of this regularization, see Nakatani [9] and Rice et al. [61]. \( f_0 \) is the steady-state friction coefficient at a reference slip rate \( V = V_0 \), \( a \) and \( b \) are nondimensional constants representing the magnitude of the direct and evolutional effect, and \( L \) is the characteristic slip displacement of the state evolution.

\[ \frac{d\theta}{dt} = \frac{1}{L} \frac{V \theta}{V} = \frac{V}{L} \left( \frac{\theta_{ss}(V)}{V} - \theta \right); \quad \theta_{ss}(V) = \frac{L}{V}. \] (28)

The geometry of the fault studied by Noda and Lapusta [5] is shown in Figure 10. 2 square patches of potentially different hydraulic diffusivity \( \alpha_{hy} \) are placed in a rate-weakening \( (a = 0.01, b = 0.014) \) region embedded in a rate-strengthening \( (a = 0.01, b = 0) \) region. The fault is loaded by a creeping at a constant rate \( V_{pl} \) near the periodic boundaries. \( \alpha_{hy} \) can be heterogeneous on natural faults depending on the local lithology as shown by experimental measurements by Tanikawa and Shimamoto [88] for the Chelungpu fault, Taiwan, a source fault of 1999 Chi-Chi earthquake. Because of restricted computational resources, Noda and Lapusta [5] used rather thick \( (w = 1 \text{ cm}) \) shear zone to make the rupture front numerically resolvable.

Distributions of cumulative slip and shear stress along the mid-depth \( (z = 0 \text{ in Figure 10}) \) is shown in Figures 11 and 12, respectively. Figure 11a, 11b, 12a, 12b represent cases with same \( \alpha_{hy} \) in both of the patches and the region between them. In these uniform cases, the earthquakes span the whole seismogenic region. This is partly because the nucleation size is not very small compared with the size of the seismogenic zone. The size of the nucleation can
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be recognized by the length of penetration of a creeping region which has high shear stress in Figure 12. Lapusta and Rice [96] showed that small nucleation size causes occurrence of frequent small earthquakes near the transition between seismogenic and aseismic regions. If the earthquake cycle is simple having only one event which spans the model, more efficient thermal pressurization causes longer recurrence interval and larger slip for each event as indicated by a spring-slider-dashpot model [97].

If there is spatial heterogeneity in the efficiency of thermal pressurization, the sequence of the earthquake becomes complex. The region with efficient dynamic weakening slips a lot when it ruptures. Thus, such a region cannot rupture in every event, and the slip deficit in the other patch is filled by more frequent and smaller events (Figure 11c). The vertical orange streaks in Figure 12c around $x = 5$ to 10 km is the stress concentration in front of the arrested earthquakes in the middle of low pre-stress region which is a result of previous earthquakes.

What is striking in Figure 12 is that the interseismic shear stress distribution is determined by the distribution of coseismic shear resistance except in the region of nucleation and around arrested rupture fronts. High velocity friction plays an important role in dynamic rupture propagation and,
Figure 11. Cumulative slip distribution at the mid-depth ($z = 0$ in Figure 10). Gray lines are plotted every 10 years, showing interseismic slip accumulation mainly outside the seismogenic region. Black lines are plotted every 1 sec during earthquakes. $\alpha_{hy-}$ and $\alpha_{hy+}$ are hydraulic diffusivities in the left and right patches, respectively. Thermal pressurization is (a) not efficient in both of the patches, (b) efficient in both of the patches, and (c) efficient only in the right patch. This figure is modified from Figure 5 and 7b in Noda and Lapusta [5].
Figure 12. Spatio-temporal distribution of shear stress at the mid-depth ($z = 0$ in Figure 10). Ambient effective normal stress is 30 MPa and $f_0 = 0.6$ with $V_0 = 10^6$ m/s so that yellow to orange color corresponds to the steady-state frictional resistance at low slip rates. Interseismic shear stress is controlled by the coseismic frictional resistance of the fault. Thermal pressurization is (a) not efficient in both of the patches, (b) efficient in both of the patches, and (c) efficient only in the right patch. This figure is modified from Figure 9 and 10b in Noda and Lapusta [5].
as its result, in determining the distribution of interseismic shear stress and thus pre-stress before the following dynamic events.

5. Summary

In this article, recent studies on the significance of high velocity friction on the dynamic rupture and its sequence are reviewed. There are multiple lines of evidences which support that the frictional resistance of a fault is much smaller than what is predicted by the Byerlee’s law and lithostatic and hydrostatic stress condition. Especially, many experimental studies suggest a remarkable weakening of a fault at coseismic slip rates. In section 3, the meaning of significant rate-weakening is clarified in terms of elastodynamics, and reviewed some of the studies on the manner (crack-like versus pulse-like) of dynamic rupture propagation in which the significance of rate-weakening plays a central role. Coseismic weakening undoubtedly affects the shear stress distribution right after an earthquake, interseismic shear stress distribution, and then the pre-stress before the following events. In section 4, a study on the effect of coseismic weakening on the sequence of earthquakes is presented which employs thermal pressurization of pore fluid. Fault constitutive law at coseismic slip rates is important not only in considering the characteristics of individual earthquake event, but also in understanding the long-term fault behavior such as the long-term shear stress under which a fault operates.

References

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