Effects of small vibrations on surface oscillation and Marangoni convection in a liquid-bridge

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Abstract

The effects of small vibrations on Marangoni convection and surface oscillation were investigated experimentally and numerically for a liquid bridge suspended between two circular disks of 7.0 mm diameter. In the experiments, a liquid bridge of 5 cSt silicone oil was formed between two circular disks in a test section and vibrated on a horizontally moving translation stage, which produced vibrations with acceleration levels of ± 10 ~ 20 mg in x, y and z directions. The critical temperature difference was
measured at the onset of oscillatory Marangoni convection for different aspect ratios, and the response of the liquid bridge surface to small vibrations was also investigated. The vibrations applied caused surface oscillations, however, no changes in the critical temperature difference were detected. The surface oscillation amplitude was also dependent on the temperature difference imposed, and a peak in the amplitude was detected at the onset of oscillatory Marangoni flow. The surface oscillation of an isothermal liquid bridge, especially the resonance vibration behavior, was also investigated numerically using a three-dimensional direct numerical simulation model based on the level set method to predict the surface oscillation. The resonance frequencies numerically predicted were in good agreement with an analytical model developed previously based on a mass-spring-damper analogy. The predicted dependence of the surface oscillation characteristics on relevant parameters is discussed in detail.

Introduction

Marangoni convection in liquid bridges has attracted a significant amount of interest in the past, mainly due to its occurrence in fabrication of semiconductor crystals of high purity by a floating zone technique both on earth and in microgravity. The microgravity environment is more advantageous as it allows formation of longer and larger liquid bridges than on the ground due to the absence of gravity. However, due to a temperature gradient imposed on the liquid bridge, thermocapillary or Marangoni convection occurs that can change from steady to oscillatory if a critical temperature gradient is exceeded. The oscillatory flow promotes non-uniformities in crystal structure such as striations [1].

The study of Marangoni convection in the float zone configuration has been simplified by the use of a half zone model. In this configuration a cylindrical liquid bridge is held between two circular rods or disks by surface tension forces. Applying a temperature difference between the two rods causes a temperature gradient, and hence a surface tension gradient, which drives the flow along the free surface and in the bulk. Previously, many experiments on Marangoni convection in liquid bridges have indicated that the critical Marangoni number, which is proportional to the temperature difference between the hot and cold disks and at which steady to oscillatory flow transition occurs, varies with the Prandtl number (Pr) of the fluid, diameter, volume ratio and aspect ratio of the liquid bridge. The critical temperature difference, $\Delta T_c$, and the frequency of temperature oscillations for liquid bridges of high temperature melts (KCl and NaNO₃) with a 6 mm diameter and various aspect ratios have been measured [2, 3].
was also suggested to be responsible for transition from steady axisymmetric flow to oscillatory flow in high Pr fluids [4].

Marangoni convection in microgravity may also be affected by small vibrations (called g-jitter) which exist on the spacecraft due to equipment operation, astronaut movements and spacecraft maneuvers. The g-jitter consists of vibrations of small amplitudes of different frequencies. There have been indications that g-jitter may cause appreciable thermal convection in space experiments [5], and thus Marangoni convection in space may also be susceptible to g-jitter. The effects of axial vibrations on the critical Marangoni number in a liquid bridge of 10 cSt silicone oil were studied on earth by applying sinusoidal vibrations with a frequency between 0.4 and 18 Hz to the liquid bridge for one aspect ratio [6, 7]. The data showed that at low vibration frequencies (f < 1.3 Hz), the critical Marangoni number, $M_{cr}$, decreased with the increasing level of vibrational acceleration, but increased with an increasing level of acceleration at higher vibration frequencies (1.3 < f < 18 Hz).

The response of the liquid bridge to high-frequency axial vibrations of the disks has also been investigated numerically [8, 9]. The disk vibration causes surface oscillations as well as mean flow due to the generation of mean vorticity in the viscous boundary layer near the disks and surface-wave propagation at a free surface.

The effect of vibrations applied to the liquid bridge in non-axial directions is less well understood. Since the liquid bridge surface could be more sensitive to transverse accelerations normal to the liquid bridge surface, the non-axial vibrations might have more significant effect on the surface oscillations and the critical Marangoni number. Thus, in this work, the effects of small amplitude vibrations with accelerations in all three directions on the onset of oscillatory Marangoni convection and free surface oscillation have been investigated experimentally.

In order to better understand the dynamic response of a liquid bridge to small vibrations under microgravity conditions, a numerical or analytical model needs to be developed. To this end, a three-dimensional numerical model has been developed using a DNS technique coupled with a level set approach to capture the interface motions of small amplitudes for a non-axisymmetric, viscous and isothermal liquid bridge. The numerical model predictions have been compared with those of a previously developed analytical model to predict the resonance behavior of a cylindrical liquid bridge subjected to horizontal vibrations. The numerical and analytical models show the functional dependence of the resonance phenomenon on the geometry and physical properties of the liquid bridge.
Experimental apparatus and instrumentation

A schematic of the test section is shown in Fig. 1. It consisted of upper and lower disks of 7.0 mm diameter made of brass. The distance between the upper and lower disks was adjustable to achieve different values of aspect ratio defined as the liquid bridge height over radius (H/R). A type-T micro-thermocouple with a wire size of 25 µm was used to detect the liquid temperature oscillations by inserting it into the liquid bridge through a hole in the upper disk. Type-T thermocouples were also used to measure the temperatures of both upper hot and lower cold disks. The details of thermocouple calibration and connection to the data acquisition system can be found elsewhere [10]. The temperatures of the upper and lower disks were controlled by passing hot and cold water from constant temperature baths through the bases to which the disks were attached. In addition, a Peltier element was attached to the lower disk so that by changing the voltage applied to the Peltier element a fine temperature adjustment for the lower disk could be achieved.

A PC-based Data Acquisition System (National Instruments Model PCI-6035E with SCXI-1102) was used to sample and record the temperature data nearly free of noise. For the liquid temperature and ΔT measurements, high-gain, low-noise DC amplifiers (NEC Model 6L06H) were used to amplify the T/C signals before the data acquisition system to maximize the signal-to-noise ratio. All the temperature data were sampled using Labview software, graphically displayed in real-time, and stored in data files.

![Figure 1. Schematic of the test section.](image-url)
In each run, a silicone oil of 5 cSt viscosity was injected between the two disks to get a volume ratio close to unity. The physical properties of 5 cSt silicon oil are shown in Table 1. The temperature difference between the two disks, ΔT, was then increased slowly by changing the voltage supplied to the Peltier element attached to the lower (cold) disk, and the temperature of the cold stream from the constant temperature bath. Fresh silicone oil was injected to overflow from the lower disk and renew the liquid bridge surface at the beginning of each experiment in order to avoid obtaining data with a contaminated liquid bridge surface.

Table 1. Physical properties of 5 cSt silicone oil.

<table>
<thead>
<tr>
<th>Property</th>
<th>5 cSt silicone oil</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ρ [kg/m³]</td>
<td>915</td>
<td>1.226</td>
</tr>
<tr>
<td>Viscosity μ [kg/ms]</td>
<td>4.575×10⁻³</td>
<td>1.78×10⁻⁵</td>
</tr>
<tr>
<td>Surface tension σ [N/m]</td>
<td>1.97×10⁻²</td>
<td></td>
</tr>
</tbody>
</table>

A PC-controlled linear stage (Parker-Daedal Model 404XR150MP) was used to impose small amplitude vibrations to the liquid bridge. The test section was mounted on the stage, which was translated horizontally with a constant speed. A video camera capturing 30 frames per second and a single-axis accelerometer were mounted on the same stage to monitor the liquid bridge surface and acceleration level, respectively. The entire experimental apparatus was mounted on a vibration-isolation optical table.

A video camera (30 frames per second) together with extension rings and a bellows provided sufficient magnification to capture and record the motion of the liquid bridge surface in the direction of the moving stage. The recorded images were captured as digital files and stored in the hard disk of a computer. A computer program developed by JAXA (formerly NASDA) was used to automatically extract the pixel-coordinates of the liquid bridge surface in different frames. A Platinum wire 125 µm in diameter was placed between the upper and lower disks in both axial and transverse directions and was recorded by the camera. These images were then used to obtain the pixel-to-micrometer conversion factors.

**Numerical simulation model**

**Equations of motion**

The equations of motion for an incompressible flow are given by the following dimensionless Navier-Stokes equations which include gravitational and small horizontal acceleration forces as well as viscosity and surface tension effects.
\begin{align}
\frac{\partial u}{\partial t} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad & (1) \\
\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + FGX + \frac{1}{\rho} \left( -\frac{\partial p}{\partial x} + \frac{\mu}{\text{Re}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{\kappa^2}{\text{We}} \delta(d) n \right) \quad & (2) \\
\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - w \frac{\partial v}{\partial z} + \frac{1}{\rho} \left( -\frac{\partial p}{\partial y} + \frac{\mu}{\text{Re}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \frac{\kappa^2}{\text{We}} \delta(d) n \right) \quad & (3) \\
\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - v \frac{\partial w}{\partial y} - w \frac{\partial w}{\partial z} + \frac{1}{\rho} \left( -\frac{\partial p}{\partial z} + \frac{\mu}{\text{Re}} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \frac{\kappa^2}{\text{We}} \delta(d) n \right). \quad & (4)
\end{align}

Here, $FGX = \frac{g_x L}{U_{\infty}^2}$ is the body force in $x$ direction induced by external acceleration, $g_x = A \omega^2 \sin \omega t$ is the external acceleration in $x$ direction, $A$ is the vibration amplitude, $\omega = 2\pi f$ is the angular frequency and $f$ is the vibration frequency, $L$ is the characteristic length. We take $L = D$, where $D$ is the diameter of the liquid bridge at top and bottom disks. $U_{\infty}$ is the characteristic velocity, $U_{\infty} = \sqrt{D g}$, where $g$ is the gravitational acceleration. $u$, $v$, and $w$ are the fluid velocities in $x$, $y$, and $z$ directions, respectively, $\rho = \rho(x,t)$ is the fluid density, $\mu = \mu(x,t)$ is the fluid viscosity, $\kappa$ is the curvature of the interface, $d$ is the normal distance to the interface, $\delta$ is the Dirac delta function, $\sigma$ is the surface tension, $n$ is the unit normal vector at the interface, $t$ is the time, $\tilde{t}$ is dimensional time, $\tilde{t} = t \sqrt{D/g}$, and $p$ is the pressure.

The dimensionless parameters used are Reynolds number, Froude number, and Weber number defined as $\text{Re} = \frac{\rho_l L U_{\infty}}{\mu_l}$, $\text{Fr} = \frac{1}{U_{\infty} \sqrt{g L}}$, $\text{We} = \frac{\rho_l L U_{\infty}^2}{\sigma}$, respectively. Here $\rho_l$ and $\mu_l$ are the dimensional liquid density and viscosity, respectively, and the dimensionless density and viscosity inside the liquid bridge are equal to 1.

Since the numerical simulation will be performed for an isothermal liquid bridge, the present model does not involve an energy equation.

**Level set function and its formulation**

The level set method was originally introduced by Osher and Sethian [11] to numerically predict the moving interface between two fluids. Instead of explicitly tracking the interface, the level set method implicitly
Vibration effects on Marangoni convection captures the interface by introducing a smooth signed distance from the interface in the entire computational domain. The level set function is taken to be positive outside the liquid bridge, zero at the interface and negative inside the liquid bridge. We consider a closed moving interface \( \Gamma(t) \), that encloses a region \( \Omega(t) \). We associate \( \Omega(t) \) with an auxiliary function \( \phi \), which is called the level set function. The level set function \( \phi \) is usually defined as the signed distance from the interface in the entire computational domain and \( \phi = 0 \) at the interface,

\[
\Gamma(t) = \{ x \in \Omega : \phi(x,t) = 0 \} .
\] (5)

We take \( \phi > 0 \) in the gas phase and \( \phi < 0 \) in the liquid phase. Therefore, we have

\[
\phi(x,t) =
\begin{cases}
> 0 & \text{if } x \in \text{Gas} \\
= 0 & \text{if } x \in \Gamma(t) \\
< 0 & \text{if } x \in \text{Liquid}
\end{cases}
\] (6)

The density \( \rho \) and viscosity \( \mu \) in the flow field can then be expressed as follows.

\[
\rho(\phi) = \rho_{\text{in}} + (\rho_{\text{out}} - \rho_{\text{in}})H_\alpha(\phi),
\] (7)

\[
\mu(\phi) = \mu_{\text{in}} + (\mu_{\text{out}} - \mu_{\text{in}})H_\alpha(\phi),
\] (8)

\[
H_\alpha(\phi) =
\begin{cases}
0 & \phi < -\alpha \\
(\phi + \alpha)/(2\alpha) + \sin(\pi\phi/\alpha)/2\pi & |\phi| \leq \alpha \\
1 & \phi > \alpha
\end{cases}
\] (9)

where the suffixes \( \text{in} \) and \( \text{out} \) stand for the liquid and gas phases, respectively, and \( \alpha \) is the prescribed thickness of the interface. In this work, we used \( \alpha = \Delta x/2 \), where \( \Delta x \) is grid spacing in \( x \) direction.

The interface motion is predicted by solving the following convection equation for the level set function of \( \phi \) given by,

\[
\phi_t + \mathbf{u} \cdot \nabla \phi = 0 .
\] (10)

The 3-D problem was solved in the following computational domain,

\[
\Omega = \{(x,y,z) | \ 0 \leq x \leq 4R, \ 0 \leq y \leq 4R, \ 0 \leq z \leq R\}.
\]
where $R$ is the radius of the liquid bridge at the top and bottom. The non-slip condition was used at the upper and lower walls in contact with the liquid bridge, and the free-slip condition was used at all other wall boundaries.

**Re-initialization of level set function**

Because we initialize the level set function $\phi$ as a signed distance from the interface, we have

$$|\nabla \phi| = 1$$

(11)

When we move the level set function $\phi$ with Eq. (10), $\phi$ will no longer be a distance function and may become irregular at later times.

$$|\nabla \phi| \neq 1$$

(12)

This will necessarily result in the variation of the interface thickness in time, making further computation and contour plotting highly inaccurate. Fortunately, we can ignore all values of $\phi$ far from the zero level set and replace the solution $\phi$ at any time by another function $\phi_0$ with the same zero set as $\phi$ and then take $\phi_0$ as the initial data to use. Sussman et al. [12] proposed an iterative procedure to fulfill the above process.

$$\phi_i = \text{sign}(\phi_0)(1 - |\nabla \phi|)$$

(13)

$$\phi(x,0) = \phi_0(x)$$

(14)

The stopping criterion for the iteration is,

$$E = \frac{\sum_{|\phi^n_{i,j,k}<\alpha|} |\phi^{n+1}_{i,j,k} - \phi^n_{i,j,k}|}{M} < (\Delta t)(\Delta x)^2$$

(15)

where, $n$ is the iteration step, $M =$ number of grid points where $|\phi^n_{i,j,k}| < \alpha$, and we take $\Delta t = \Delta x / 10$. The above procedure was followed to re-initialize the level set function.

**Improved re-initialization of level set function**

By carrying out re-initialization, the level set function will remain a distance function at later times, and will ensure the interface to have a finite thickness all the time. Another important issue is mass conservation. For incompressible flows, the total mass must be conserved in time. However, just like other Eulerian methods, such as the VOF method, even with the
above re-initialization procedure, it has been found that the total mass is not completely conserved in time. To overcome this difficulty, an improvement has been made in the re-initialization procedure aimed at preserving the total mass in time. We classify the grid points into two groups: the first group includes the grid points that are the closest to the interface and on the interface, and the other grid points belong to the second group. For simplicity, let us consider a 2-D domain. If any one of the following conditions is satisfied, \((i, j)\) belongs to the first group points, otherwise, \((i, j)\) belongs to the second group points.

\[
\begin{align*}
&\left(\phi_{i,j}\right)\left(\phi_{i,j+1}\right) \leq 0, \\
&\left(\phi_{i,j}\right)\left(\phi_{i,j-1}\right) \leq 0, \\
&\left(\phi_{i,j}\right)\left(\phi_{i+1,j}\right) \leq 0, \\
&\left(\phi_{i,j}\right)\left(\phi_{i-1,j}\right) \leq 0,
\end{align*}
\]

For the first group points, we solve the convection equation \(\phi_t + \mathbf{u} \cdot \nabla \phi = 0\) and denote the updated \(\phi\) by \(\phi^{(n+1)}\). We have now advanced one time step. The zero level set of \(\phi^{n+1}\) gives the new interface position and \(\phi^{n+1}\) is a distance function. For the second group points, we solve the convection equation \(\phi_t + \mathbf{u} \cdot \nabla \phi = 0\) and denote the updated \(\phi\) by \(\phi^{(n+1/2)}\), and then construct a new distance function by solving \(\phi_t = S\left(\phi^{(n+1/2)}\right)(1-|\nabla \phi|)\) with \(\phi(x,0) = \phi^{(n+1/2)}(x)\) to steady state. We denote the steady state solution by \(\phi^{n+1}\) and we have now advanced one time step. The zero level set of \(\phi^{n+1}\) gives the new interface position and \(\phi^{n+1}\) is a distance function.

**Body force due to surface tension force**

The model of Continuum Surface Force (CSF) was employed to treat the surface tension force at an interface, which interprets the surface tension force as a continuous effect across the interface rather than as a boundary condition on the interface [13]. By using the level set function, body force due to surface tension can be expressed as,

\[
\frac{1}{We} \kappa \delta (d) \mathbf{n} = \frac{1}{We} \kappa (\phi) \delta (\phi) \nabla \phi
\]
The curvature of the interface is evaluated from

\[ \kappa(\phi) = -(\nabla \cdot \mathbf{n}) = -\nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \]  \hspace{1cm} (21)

The Dirac delta function is defined as

\[ \delta_\alpha(\phi) = \frac{dH_\alpha(\phi)}{d\phi} = \begin{cases} 0 & |\phi| > \alpha \\ \frac{1}{2\alpha} \left[ 1 + \cos \left( \frac{\pi \phi}{\alpha} \right) \right] & |\phi| \leq \alpha \end{cases} \] \hspace{1cm} (22)

**Poisson equation solver**

Briefly, we may write the Poisson equation for pressure as

\[ \nabla \cdot \left( \frac{\nabla p}{\rho} \right) = \frac{\nabla \mathbf{G}}{\Delta t} \] \hspace{1cm} (23)

\[ \mathbf{G} = \begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} = u + \Delta t \left( - \left( \mathbf{u} \cdot \nabla \right) \mathbf{u} + \frac{n_{i(i=x,z)}}{Fr_{i(i=x,z)}} + \frac{1}{\rho} \nabla \left( 2 \mu \bar{D} \right) + \frac{1}{We} \kappa(\phi) \delta(\phi) \nabla \phi \right) \] \hspace{1cm} (24)

where \( n_{i(i=x,z)} \) is a unit normal vector in \( x \) or \( z \) direction, \( \bar{D} \) is the viscous stress tensor.

The Successive Over Relaxation (SOR) method has been used to solve the Poisson equation for pressure.

**Algorithm to catch the small displacement of the surface**

When a small vibration is applied, the movement of the liquid bridge surface can be extremely small, in the order of microns. It is not easy to capture such tiny surface movements in gas-liquid two phase flow simulations using grids of much larger dimensions. The Volume of Fluid method would have an insurmountable difficulty in overcoming this spatial resolution problem. In contrast, the level set approach can avoid this problem using the following new algorithm developed in this work and described below:

Step 1. Determine a monitoring point on the interface.
Step 2. Find out the grid point, which bounds the monitoring point outside the interface or on the interface when the interface moves through the grid at every time step.
Step 3. Calculate the position of the monitoring point using the information of the grid point found out in step 2.

Let us take a cross section in the $x$-$y$ plane at a certain height of the liquid bridge. When the interface traverses through the grid, there are two ways for the interface to intersect with the grid. One way is that the interface intersects with the grid line between two grid points as shown in Fig. 2, and the other is that the interface directly falls on one of the grid points. In Fig. 2, $\Gamma$ represents the interface, and $(i, j, k)$ and $(i+1, j, k)$ represent two arbitrary grid points on opposite sides of the interface ($\Gamma$). The level set function $\phi$ can be constructed by choosing the sign of $\phi$ to be negative inside the interface and positive outside the interface. If the interface intersects with the grid point, for example, $(i, j, k)$, the level set function $\phi$ at this point should be zero and the position of the monitoring point in the $x$ direction can be determined by using the coordinate value of $(i, j, k)$. If the interface intersects with the grid line between the two grid points, for example, $(i, j, k)$ and $(i+1, j, k)$, the two points should be determined as follows:

$$\phi(i, j, k)\phi(i+1, j, k) < 0$$

(25)

According to the principle of the level set method, the value of the level set function $\phi$ at $(i, j, k)$ is equal to the signed distance from this point to the interface. Therefore, the position of the monitoring point in the $x$ direction can be determined by using the coordinate value of $(i, j, k)$ and the value of the level set function $\phi$ at $(i, j, k)$.

![Figure 2. An interface intersecting with a grid at x-y plane.](image)

Grid independence studies conducted showed that the surface amplitude predicted using the $51\times51\times51$ grid was fairly close to the one predicted using the $101\times101\times51$ grid, while the $41\times41\times51$ grid and the $31\times31\times51$ grid predictions displayed significant differences. The $51\times51\times51$ grid was therefore adopted for further simulations in this work, as the use of finer grids such as the $101\times101\times51$ grid or the $71\times71\times51$ grid required excessive computational times. The time step $\Delta t$ was determined by the restrictions due to the CFL condition, gravity, external acceleration,
viscosity, and surface tension, in a manner similar to that proposed by Sussman et al. [12].

Results and discussion

Onset of oscillatory Marangoni convection

Figure 3 shows the liquid temperature oscillation data, their root mean square (RMS) values and the disk temperature difference, $\Delta T$, for a silicone oil bridge of 7.0 mm diameter with an aspect ratio (height/radius) of 0.69 without any vibrations applied. At low temperature differences, the liquid temperature showed only small fluctuations (noise), but the amplitude of the temperature oscillations increased sharply at a certain disk temperature difference. This is clearly shown in Fig. 3(b) by a sharp increase in the slope of the RMS value at

![Figure 3](image)

**Figure 3.** Detection of the onset of oscillatory Marangoni convection for the case of no vibration applied to a liquid bridge: (a) fluid temperature oscillations, (b) RMS of fluid temperature oscillations, and (c) temperature difference between upper and lower disks.
the critical temperature difference, $\Delta T_{cr} = 45.3 \sim 45.5 \degree C$. The power spectrum density of the liquid temperature oscillations showed no peaks for steady Marangoni convection in the liquid bridge ($\Delta T < \Delta T_{cr}$), while a peak appeared at 1.4 Hz for the oscillatory flow case when the temperature difference exceeded the critical value.

For the vibration tests, the acceleration conditions for the test section mounted on a translation stage along with a sensitive accelerometer were first determined for all three directions (x = direction of the stage movement, y = horizontal direction normal to the stage movement, and z = vertical direction). When the stage was moving with a constant speed, the mechanically driven translation stage containing a servo motor and a ball screw mechanism produced small accelerations in all three directions as shown in Fig. 4.

![Figure 4. Acceleration levels in x, y and z directions measured on a moving stage.](image)
While the translation stage was moving with a constant speed, the lower disk temperature was decreased by applying an increasing voltage to the Peltier element. Figure 5 shows the liquid temperature oscillations, their RMS values and the temperature difference between the two disks for this case. The transition from steady to oscillatory Marangoni convection occurred at $\Delta T_{cr} = 45.1$ °C, which is practically unchanged from the value obtained for the stationary test section. The power spectrum density of the liquid temperature data showed no peak for steady Marangoni convection while a peak appeared at 1.4 Hz for the oscillatory flow case. This frequency is the same as that obtained for the non-vibrated liquid bridge, thus the oscillation frequency remained unaffected by the small vibrations applied by the moving stage.

![Figure 5](image.png)

**Figure 5.** Onset of oscillatory Marangoni convection for the case of small vibrations applied to the liquid bridge: (a) fluid temperature oscillations, (b) RMS of fluid temperature oscillations, and (c) temperature difference between upper and lower disks.
A series of experiments were conducted for silicone oil bridges with a diameter of 7.0 mm and different heights or aspect ratios. The critical temperature difference was measured in each experiment and the results are shown in Fig. 6 for five aspect ratios. It can be concluded from this figure that small vibrations shown in Fig. 4 had no significant effect on the critical temperature difference for the range of aspect ratios studied (0.68 ~ 1.03). As will be presented and discussed in the next section, the liquid bridge surface responded to the applied vibrations by oscillating with significant amplitudes, especially at the onset of oscillatory Marangoni convection. However, the temperature difference required for the onset of oscillatory Marangoni convection was unaffected by the applied vibrations as shown in Fig. 6. This may be due to the fact that the directions of surface oscillation and surface tension-driven flow are normal to each other, and thus the axial fluid flow is not directly affected by the transverse surface oscillations.

Figure 6. Effect of small vibrations on the critical temperature difference at the onset of oscillatory Marangoni convection in liquid bridges of different aspect ratios.

**Liquid bridge surface oscillations**

The effect of small vibrations on the liquid bridge surface was also investigated. Although not presented here, when there were no external vibrations applied, the surface of the liquid bridge did not show any displacement detectable with the resolution of the video camera system used in this work. On the other hand, the liquid bridge surface responded to small vibrations with oscillations of varying amplitudes depending on the
temperature difference between the upper and lower disks. A consecutive sequence of instantaneous axial profiles of the liquid bridge surface recorded every 1/30 second under small vibrations and applied temperature difference close to the critical $\Delta T$ showed the surface oscillation amplitude increased with the distance below the hot corner ($z = 0$).

In Fig. 7, the peak-to-peak amplitudes of the oscillating liquid bridge surface are plotted for different disk temperature differences. In the absence of any temperature difference, $\Delta T = 0$, the peak-to-peak oscillation amplitude increased from about 20 $\mu$m at a distance of 100 $\mu$m below the hot corner to 50 $\mu$m at a distance of 500 $\mu$m below. By maintaining the upper disk temperature constant and decreasing the lower disk temperature, the surface temperature of the liquid bridge below the hot corner also decreased and surface tension generally increased over the entire liquid bridge surface. This increase in surface tension had a stabilizing effect on the oscillation of the liquid bridge surface, therefore the peak-to-peak amplitude decreased with increasing $\Delta T$ as long as $\Delta T$ was less than $\Delta T_{cr}$.

**Figure 7.** Peak-to-peak amplitudes of liquid bridge surface oscillations for the case of small vibrations applied to a 5 cSt silicone oil liquid bridge.
In Fig. 7, the peak-to-peak amplitude for $0 < \Delta T < \Delta T_{cr}$ was reduced to about a half of that obtained for $\Delta T = 0$. The stabilizing effect was more pronounced further below the hot corner where the local temperature was lower and surface tension higher.

Increasing $\Delta T$ to the critical value where transition from steady to oscillatory Marangoni convection occurred caused an abrupt increase in the peak-to-peak amplitude although the average temperature of the liquid bridge was lower and therefore the stabilizing effect of surface tension was greater. Two different measurements at the onset of oscillatory flow shown in Fig. 7 indicate that the amplitudes of surface oscillations to be nearly twice and four times as large as those obtained at $\Delta T = 0$ and $\Delta T < \Delta T_{cr}$, respectively. Interestingly, further increasing the temperature difference above the critical value, $\Delta T > \Delta T_{cr}$, caused the peak-to-peak surface oscillation amplitude below the hot corner to significantly decrease, although it did not change near the hot corner within about 30 $\mu$m below.

The reason for the peculiar dependence of the surface oscillation amplitude on the imposed temperature difference described above is not clear. The mechanism of transfer and conversion of vibration energy to surface oscillation as well as flow/temperature oscillation at the onset of oscillatory Marangoni convection needs to be investigated further in the future.

**Numerical simulation of surface oscillation characteristics**

The initial shape of the isothermal liquid bridge simulated under normal gravity is shown in Fig. 8, which is a straight cylinder, and the diameter and height of the liquid bridge were 7.0mm and 3.5mm, respectively. Both the liquid bridge and the surrounding air are at rest initially. After the computation is started, the liquid bridge shape first deforms due to gravity and then responds to the small vibrations applied in the horizontal direction. The amplitudes of periodic surface oscillations predicted were determined at different elevations above the bottom of the liquid bridge.

The evolution of the liquid bridge shape due to horizontal vibrations at an acceleration level of 20 mg is illustrated in Fig. 9(a)-(b). These figures show the liquid bridge shape as viewed at an angle of 30 degrees from the ground. The inward and outward deformations near the upper and lower disks are consistent with the increase and decrease in the hydrostatic pressure near the lower disk and upper disk, respectively, due to the effect of gravity. Because the applied vibration in the horizontal direction is so small, the shapes appear to be the same between Figs. 9(a) and (b), however, the surface oscillation is predicted and can be studied quantitatively as described below.
Figure 8. Initial shape of a 3-D liquid bridge.

(a) $t = 0.0267$ s

(b) $t = 0.1015$ s

Figure 9. Evolution of a free surface of a liquid bridge oscillating due to a small horizontal vibration under normal gravity.
To quantify the response of the liquid bridge surface to small vibrations applied in the horizontal direction, simulations were systematically conducted for a 5 cSt silicone oil bridge first under normal gravity. The diameter of the liquid bridge was 7.0 mm, and the height was 3.5 mm. The applied vibration frequencies ranged from 3 to 17 Hz, in increments of 2 Hz. The acceleration level in the horizontal direction was kept constant at 18-mg. Three monitoring points on the surface of the liquid bridge were selected at heights of $H/4$, $H/2$, $3H/4$, from the bottom disk. Figure 10 shows the oscillations of the surface for vibration frequencies of 3 and 9 Hz. Because of the deformation of the liquid bridge due to gravity, the surface oscillation amplitudes obtained at the three heights are different, and the mid-height of

![Figure 10](image_url)

**Figure 10.** Evolution of the surface positions at three different heights for an applied vibration frequency of (a) 3.0 Hz and (b) 9.0 Hz.
the bridge is the position that gives the largest amplitude among the three monitoring points. The tendency of the surface oscillation at heights $H/4$, $H/2$, and $3H/4$ is the same except for the initial period.

The relationship between the surface amplitude at the mid-height ($z = H/2$) and the applied vibration frequency is shown in Fig. 11 for the case with the ratio of the minimum diameter of the liquid bridge to the disk diameter ($D_{\text{min}}/D_{\text{disk}}$) being equal to 0.778. In Fig. 11, an extremely large amplitude can be seen at about 11Hz. The amplitudes at other vibration frequencies are low, and they have the same order of magnitude. It is clear that the resonance frequency of this liquid bridge is located at about 11Hz.

![Figure 11](image)

*Figure 11. Relation between surface oscillation amplitudes and applied vibration frequencies for a liquid bridge ($D = 7.0 \text{ mm}$, $H = 3.5 \text{ mm}$) under normal gravity.*

**Surface oscillation characteristics under zero gravity**

An analytical model was previously developed by Ichikawa et al. [14] for a liquid bridge with a shape of a straight cylinder under zero gravity based on a spring-mass-damper system. The following expression was derived for the resonance frequency.

$$f_{\text{Res.}} = \frac{\omega_{\text{Res.}}}{2\pi} = \frac{1}{2\pi H} \sqrt{\frac{2}{B} \left( \frac{8\sigma}{D\rho} - \frac{v^2}{H^2 B} \right)}$$

(26)

The parameter, $B$, represents the fraction of total liquid mass which actually moves in response to an external vibration.

To compare the numerically predicted resonance frequency with the analytical model, additional numerical simulations were conducted for a liquid bridge under zero gravity. In this set of computations, the diameter
and height of the liquid bridge were 8.4mm and 4.2mm, respectively, and the applied vibration frequency ranged from 3 to 17Hz. The acceleration level in the horizontal direction was again kept constant at 18-mg.

Figure 12 shows the relation between the predicted surface oscillation amplitude and the vibration frequency applied. The surface oscillation amplitude is seen to remain low at all vibration frequencies except for 9Hz, at which significantly larger amplitude is obtained due to the resonance behavior of the liquid bridge.

![Figure 12](image1.png)

**Figure 12.** Relation between surface oscillation amplitude and vibration frequency for a liquid bridge (D = 8.4 mm, H = 4.2 mm) under zero gravity.

**Effects of liquid bridge diameter and height on the resonance frequency**

Figure 13 shows the relation between the numerically predicted resonance frequency and the disk diameter for a liquid bridge of H=3.5mm. The numerical simulations were performed at an acceleration level of 5mg in the absence of gravity. As predicted by the analytical model referred to as theory in Fig. 13, the resonance frequency decreases with the increasing disk diameter. The predictions of the analytical model are also shown in the figure for several different values of $B$, $B=1$ and $B = 0.9$ (for $D = 5.0$ mm) or 0.92 (for $D = 7.0$ and 10.0 mm). In general, due to the no slip condition at the upper and lower disk surfaces, the moving mass should not be equal to the total mass of the liquid, thus $B$ can not be equal to 1.0.

For the analytical model, the value of $B$ is expected to lie between 0.5 and 1.0. By comparing the numerical results with the analytical model predictions, it is clear that the analytical predictions with $B=0.9$ or 0.92 are
closer to the numerical results than those obtained with $B=1.0$. That means about 90% of the mass of the liquid bridge is induced to move in response to the horizontal vibrations.

Next, the effect of the liquid bridge height on the resonance frequency is investigated. Three different liquid bridge heights were considered while keeping the diameter of the bridge constant at $D=7.0$ mm and the acceleration at 5mg in the absence of gravity. As shown in Fig. 14, the resonance frequency

![Figure 13. Effect of disk diameter on resonance frequency of a liquid bridge.](image)

![Figure 14. Effect of liquid bridge height on resonance frequency (D = 7.0 mm).](image)
decreases with the increasing height of the liquid bridge. The resonance frequency varies over the range of 3-11Hz when the height of liquid bridge is changed from 3.5mm to 10.5mm. Both the computational results and the analytical model predictions show the same trend, and they agree closely for different values of the parameter $B$.

**Effects of physical properties on the resonance frequency**

The liquid bridge is supported between the upper and lower disks by surface tension force. Therefore, varying the surface tension should affect the vibration characteristics of the liquid bridge and its resonance frequency. Figure 15 shows the relation between the resonance frequency and surface tension predicted for a liquid bridge of $D = 7.0$ mm and $H = 3.5$ mm, when the acceleration level was kept at 5mg in the absence of gravity. The resonance frequency is seen to increase with the increasing surface tension. For example, when the surface tension is increased from $1.97 \times 10^{-2}$ N/m to $7.2 \times 10^{-2}$N/m, the resonance frequency increases from 10.6Hz to 20.6Hz. Increasing the surface tension would make the liquid bridge less flexible and the resonance frequency would increase.

![Figure 15. Effect of surface tension on the resonance frequency of a liquid bridge (D = 7.0 mm, H = 3.5 mm).](image)

Figure 16 shows the relation between the resonance frequency and density of the liquid bridge predicted for a liquid bridge of $D = 7.0$ mm and $H = 3.5$ mm, at an acceleration level of 5mg in the absence of gravity. The density of the liquid bridge was changed from 700 kg/m$^3$ to 1200 kg/m$^3$. It is clear from Fig. 16 that the resonance frequency decreases with the increasing density.
The effects of both the surface tension and liquid density predicted by the numerical model are in close agreement with the analytical model predictions. The best agreement is obtained with $B = 0.9 \sim 0.92$ rather than $B = 1.0$ in the analytical model.

**Optimum value of $B$**

In the analytical model, a parameter $B$ was introduced representing the ratio of the moving mass to the total mass of the liquid bridge. Its value would depend on the height of the liquid bridge for a given diameter, or the aspect ratio. From several comparisons of analytical model predictions with the numerical simulations, it is clear that the best value of $B$ is not constant for different aspect ratios of the liquid bridge. The relation between the best value of $B$ and the aspect ratio has been determined and is shown in Fig. 17. Because of the no-slip condition at the upper and lower disk surfaces, an increasingly smaller fraction of the total liquid mass would respond to the external vibrations, if the liquid bridge height is decreased. Thus, it is reasonable that the best optimum value of $B$ increases as the aspect ratio is increased, reaching a constant value almost equal to unity for aspect ratios greater than 1.5.

Using a value of $B = 0.9$, the resonance frequencies of a liquid bridge (5 cSt silicone oil) predicted by Ichikawa et al.’s [14] analytical model are shown in Fig. 18. The resonance frequency is predicted to decrease rapidly with the increasing liquid bridge diameter and aspect ratio.
The effects of small vibrations on the transition from steady to oscillatory Marangoni convection and surface oscillation characteristics in liquid bridges of silicone oil (5 cSt) were experimentally and numerically investigated for a diameter of 7.0 mm and different aspect ratios. Small vibrations were found to significantly affect the transition to oscillatory convection, with a critical moving volume ratio $B$ and aspect ratio of the liquid bridge. The resonance frequencies of the liquid bridge were also predicted using Ichikawa et al.’s [14] analytical model with $B = 0.9$.

**Figure 17.** Relation between the moving volume ratio $B$ and aspect ratio of a liquid bridge.

**Figure 18.** Resonance frequencies of a liquid bridge (5 cSt silicone oil) predicted by Ichikawa et al.’s [14] analytical model with $B = 0.9$.

**Conclusions**

The effects of small vibrations on the transition from steady to oscillatory Marangoni convection and surface oscillation characteristics in liquid bridges of silicone oil (5 cSt) were experimentally and numerically investigated for a diameter of 7.0 mm and different aspect ratios. Small
external vibrations of about ±10 ~ 20 mg applied to the liquid bridge in all three directions were found to exert little influence on the onset of oscillatory Marangoni convection for aspect ratios ranging from 0.68 to 1.03, however, the surface of the liquid bridge showed significant oscillations even at small vibration levels. The vibration-induced surface oscillations decreased in amplitude as the temperature difference was raised by keeping the upper disk temperature constant and decreasing the lower disk temperature. The consequent increase in surface tension reduced the surface oscillation amplitude, however, an abrupt increase in the amplitude was obtained at the onset of oscillatory Marangoni convection. Increasing the temperature difference further in the liquid bridge resulted in a reduction in the surface oscillation amplitude.

The vibration-induced oscillation of an isothermal liquid bridge was also investigated numerically and the predictions were compared with an analytical model previously developed based on a mass-spring-damper analogy. The surface oscillation of a cylindrical liquid bridge induced by a horizontal vibration applied with a given frequency and acceleration level was predicted by a three-dimensional DNS analysis with a level set approach to capture the microscopic surface oscillations. The existence of a resonance frequency, at which the oscillation amplitude of the liquid bridge increases significantly, was clearly predicted both numerically and analytically. The numerical and analytical model predictions were consistent with one another.

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References